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Direct and Inverse Cinematic Resolution Calculation using Denavit-Hartenberg Algorithm and Geometric Method for Robot with 5 DOF

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Abstract*– This manuscript aims to shine light on the analysis done for the direct and inverse kinematics of a 5 DOF robot. Where the main objective is to add a 5th degree of freedom to a traditional SCARA robot due to the extent of the study, it was fitting to cover in a more in-depth manner the control system separately to continue to add to the state of the art. It was possible to solve the direct and inverse kinematics of an inverse SCARA robot with 5 DOF using Denavit-Hartenberg algorithm and geometric method, respectively. The Denavit-Hartenberg algorithm solves kinematics based on homogenous transformation matrices to pass through one system to another using the coordinates systems of every joint and link. Geometric method allows to solve complex kinematics in a simplified way by considering just the coordinates that provides finite solutions. A successful control system for an inverse SCARA of 5 DOF could be built using these two methods.

***Keywords*—direct kinematics, inverse kinematics, Denavit-Hartenberg algorithm, control system.**

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I. Introduction

This manuscript aims to shine light on the analysis done for the direct and inverse kinematics of the robot designed and constructed in [1] adding an inverse component to one of its joints. Where the main objective was to add a 5th degree of freedom to a traditional SCARA robot due to the extent of the study, it was fitting to cover in a more in-depth manner the calculations and an evaluation of a change in the joint structure separately to continue to add to the state of the art. This effort allows for others to continue to study in this line of investigation and continue to add value to the field of robots in general as well as the industry and its process.

II. Literature Review

Throughout the years various studies and research in areas of kinematics in robotics have been done. Serrezela et al. in 2017 through their work [2] utilized a 3 degree of freedom robot prototype on which the kinematic analysis was carried out, to complete the analysis process an application was used for the control of a clamp. The app allowed via a program the translation and rotation of the claim to comply with the desired position. Das & Dulger in 2004 [3] in areas of mathematical modelling carried out a full development for a Scara robot, development which included dynamics considering servo actuation using Lagrangian mechanics to derive the equation of motion.

Cantarero & Reyes, 2021 [4] made use of the spiral method in order to perform the development of a mathematical model used for the definition of the Scara robots control from a kinematic point of view. The previously mentioned control kinematics were obtained through the calculation forward and also inverse kinematic principles of analysis. Applying the Denavit – Hartenberg (D-H) algorithm the authors were able to describe the orientation and position of the robot. Additionally for the trajectory tracking the use of polynomial interpolators is utilized. Ordóñez-Avila et al, 2022 [5] performed a comparison between 2 manipulator developments used for industrial tasks taking advantage of the CoppeliaSim software. A newly designed 7 degree of freedom manipulator by means of the D-H algorithm. The main aim of this study was to determine if the addition of 1 DOF would interfere with the operability of the robot in working conditions. Vlădăreanu et al, 2012 [6] develops a control system of dynamic nature applied to a walking robot. The robot is described as hexapod where the distribution for its degrees of freedom is divide into to main groups. 3 of the DOF are destined for the positioning movement, establishing one for each leg. The other 3 DOF belong to the orientation of the paw foot.

Wen et al, 2014 [7] apply D-H for the calculation of the forward kinematics of a humanoid robot called “NAO” obtaining via this process the analysis necessary for its control. Azad et al, 2019 [8] provides a full dynamic equation for the relationship between the kinematics of a robot for its movements. Initially the screw theory is used given the upside it has in terms formula compactness when calculating the Jacobian matrices is called for and making it easy to apply the principle of virtual work. Subsequently, having calculated the matrices used for the relationship between input and output data, links, and manipulator; the final equation defined. Katta et al, 2021 [9] present a method involving computer vision approaches and On-body or Aruco markers in hand with the kinematics for the pose estimation of a 5 DOF Robotic arm, the PhantomX Reactor. Additionally, the pose of a camara attached to a manipulator is estimated in relationship to an object that is targeted. Zheng et al, 2022 [10] applies the GCOP (general constrained optimization problem) in synchrony with the SQP (sequential quadratic programming) feed with a sensor input for a dynamic algorithm used in path planning process for a robot. The mentioned algorithm is an integral part in the functioning of a turtle inspired robot capable of avoiding obstacles and planning travel paths.

Huczala et al, 2021 [11] offers their finding in form of four algorithms for pose estimation all having as a base the Denavit-Hartenberg convention. All the algorithms aim to find an optimal design for local search optimization having as input the system a kinematic structure. Han et al, 2021 [12], implemented offset RR joints that offer high levels of precision, in areas of load to size ratios very high values and equally high offerings in regards to stiffness. All the features mentioned aimed to be used in the development of a parallel platform. Also, in this study a development of a system with zero gap bead shaft is implemented for the correct and positive adjustment of the complete mechanism overall accuracy and stiffness. In 2020, Zhao et al [13] present the development of a serial industrial robot in which the D-H algorithm for kinematics is used aiming to and successfully establishing the home position and tool coordinates error models for the 6 DOF robot. Having the error model, a calibration method is introduced based on fix constrains.

Shah et al, 2012 [14] attempted to correlate the Euler angles and Denavit-Hartenberg (D-H) parameters through a D-H parametrization performed on the Euler angles. In addition, the authors define the method for the acquisition of the D-H parameters for any value of Euler angles established. Corke, 2002 [15] presented a serial link type robot for which an approach to determine parameter for its kinematics is offered. Using the D-H notation a corpus of results was available including algorithms for dynamics, kinematics, planning for motion and code implementations. Singh et al, 2020 [16] design and developed mobile robot equipped with a differential drive. Experimentation and evaluation of the robot was carried out in an environment free of obstacles. The robot presented is equipped with position estimation features and the capacity to move on a 2d plane in any direction by way of speed control of driving wheels. To complete current state estimation and goal position, forward and inverse kinematics are applied.

Petrescu & Petrescu, 2016 [17] solved and presented the kinematics method applied to the position estimation of a robotic structure. The authors also provide insight into the kinematics solved as for inverse and direct models. Trujillo et al, 2018 [18] studied the industrial controller CompactRIO in order to develop an algorithm for generating trajectories. The developed algorithms offer the possibility to control various parameters of a 5 DOF robot such as the artistic movement, linear movement and arc. Ciornei et al, 2018 [19] obtained the dependency of two shafts in terms of their movements. The D-H algorithm was performed to obtain the kinematics for the shaft joints decoupling. This study is performed justified by the fact that the shafts present hair pair joint categorized under those of point surface. He et al, 2017 [20] developed a SCARA robot for application of the handling of ceramic substrate and wafers aiming to add higher levels of flexibility to the system as a whole. Ibrahim & Khalil, 2010 [21] study robots with modules that are connected serial and parallelly with non-redundancy, known as Hybrid robots. The study proposes a method that generalizes the Newton–Euler algorithms which has been developed for serial robots with expansions the inverse and direct kinematics. Saraf et al., 2021 [22] utilized MATLAB and implemented the D-H algorithm's parameters for the modelling process of a 6 DOF robotic arm driven by a hydraulic system. Mocnik et al, 2022 [23] propose a method which transforms videos captures of gestures and stores them as 3D models in the repository of a conversational agent to be used for its motor skills. Ciorneu et al, 2019 [24] offered a method applied to spatial mechanisms in which the kinematics are studied and solved. The method presented bases itself on the D-H system in areas of the coordinate frames that are attached to a mechanisms element and in areas of kinematic chains the closure of the matrix equation.

The remainder of the study is presented as follows:

- Section III presents the methods applied during the design and parametrization of the model developed for the mathematical standpoint.
- In Section IV an insight and thorough presentation of the results is given.
- Section V summarized the study's most important and valuable findings and their relation to the aim of the research performed.

III. Methods

A. Direct Kinematics

For direct kinematics, the Denavit - Hartenberg algorithm was used as a resolution method. For this method, it is needed to define the coordinate systems and number of systems for each link and joint. The number of systems is determined by the degree of freedom of the robot. Once the coordinate systems associated to each link have been established it is possible to move from one link to the next one by means of four homogeneous matrix transformations, which are: rotation in Z-axis (θ), translation along the Z-axis (d), translation along the X-axis (a) and rotation in the X-axis (α) [25]. To obtain each transformation matrix, Table I must be filled in, where the columns represent the transformations, and the rows represent the number of systems. From the parameters of Table I the homogeneous transformation matrices, that describes the passage from system i-1 to system i, can be defined. These parameters must be substituted in the Denavit - Hartenberg base matrix H (1) (2).

TABLE I
Parameters Denavit – Hartenberg (D-H)

i	$\boldsymbol{\theta}$	**D**	**a**	**α**
i+1				

$$H_{(i-1)\to i} = Rotz(\theta_i)\cdot Tz(0,0,d_i)\cdot Tx(a_i,0,0)\cdot Rotx(\alpha_1) \quad (1)$$

$$H_{(i-1)\to i} = \begin{pmatrix} \cos\theta_i & -\cos\alpha_{i^*}\sin\theta_i & \sin\alpha_{i^*}\sin\theta_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\alpha_{i^*}\cos\theta_i & -\sin\alpha_i * cos\theta_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

B. Inverse Kinematics

The determination of joint variables is known as inverse kinematics. This process is carried out with respect to specifically the position and orientation the end effector has. Compared to direct analysis, the inverse analysis presents itself with higher levels of complexity. The reason it is more complex is because it is not always possible to find an explicit solution as the equations are nonlinear, multiple solutions or infinite solutions may exist, and due to the architecture of the robot, there may be no admissible solutions. Two methods are commonly used to solve inverse kinematics: algebraic and geometric method. In this research, the geometric method is used due to its accuracy since the angles direction of the robot are known and considerations to position the end effector are made only for the necessary degrees of freedom [26].

The geometric method consists of finding enough geometric relationships involving the coordinates of the joints and the end effector. A diagram of its links is made with a representation of their sizes and trigonometric relations are obtained through different identities to find the values of the relevant joints that are considered to position the end effector of the robot [27].

IV. Results and Analysis

A. Direct Kinematics

A diagram of the robot was done in which it indicates the name and coordinate systems for each link and joint of the

SCARA of 5 DOF, Fig. 1. The Z axes were arbitrarily located, the X axes were positioned in accordance with the right hand rule between the Z axes, and the Y axes along the common normal axe. System 0 corresponds to joint 1 and system 5 corresponds to the final effector.

To describe the passage from coordinate system i-1 to the onto current system known as "i" through the transformation matrices, four Denavit -Hartenberg parameters in Z-rotation, X-rotation, X-translation, and Z-translation, were determined in Table II. These parameters were substituted in (2) to obtain the homogeneous transformation matrices of the system 0 to 1 (3), 1 to 2 (4), 2 to 3 (5), 3 to 4 (6) and 4 to 5 (7). The product of the multiplication of these matrices shows the movement needed to go from the coordinate system 0 to 5, which is represented by (8).

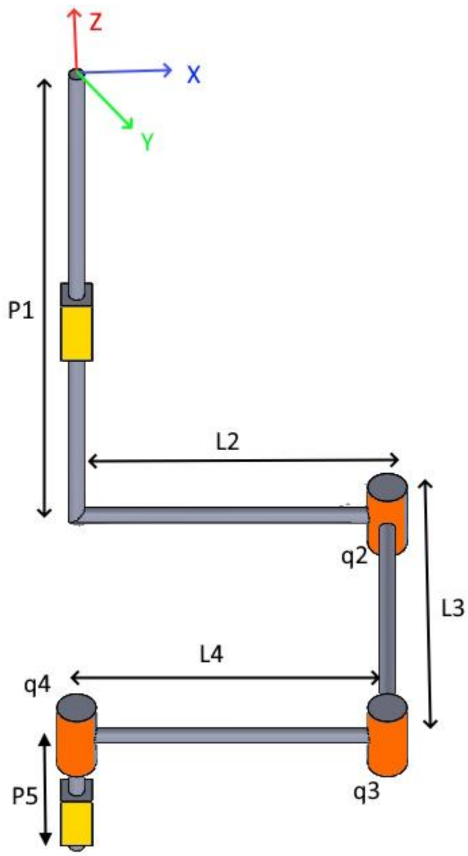

Fig. 1 Diagram of coordinates systems of the Inverted SCARA robot of 5 DOF.

TABLE II
D-H PARAMETERS FOR AN INVERTED SCARA OF 5 DOF

i	θ	d	a	α
1	0	-P1	L2	0
2	-180+θ1	0	L3	0
3	-180+θ2	0	L4	0
4	θ3	0	0	0
5	0	-P5	0	0

$$H_{01} = \begin{pmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -P_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

$$H_{12} = \begin{pmatrix} \cos(\theta_1 - 180) & -\sin(\theta_1 - 180) & 0 & L_3 * \cos(\theta_1 - 180) \\ \sin(\theta_1 - 180) & \cos(\theta_1 - 180) & 0 & L_3 * \sin(\theta_1 - 180) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

$$H_{23} = \begin{pmatrix} \cos(\theta_2 - 180) & -\sin(\theta_2 - 180) & 0 & L_4 * \cos(\theta_2 - 180) \\ \sin(\theta_2 - 180) & \cos(\theta_2 - 180) & 0 & L_4 * \sin(\theta_2 - 180) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

$$H_{34} = \begin{pmatrix} \cos(\theta_3) & -sin(\theta_3) & 0 & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & -L_6 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{7}$$

$$H_{45} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -P_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6}$$

$$H_{05} = \begin{pmatrix} \cos(\theta_1 + \theta_2 + \theta_3 - 360) & -\sin(\theta_1 + \theta_2 + \theta_3 - 360) & \dots \\ \sin(\theta_1 + \theta_2 + \theta_3 - 360) & \cos(\theta_1 + \theta_2 + \theta_3 - 360) & \dots \\ 0 & 0 & \\ 0 & 0 & \end{pmatrix}$$

$$\begin{pmatrix} \dots & L_2 + L_4 * \cos(\theta_1 + \theta_2 - 360) + L_3 * \cos(\theta_1 - 180) \\ \dots & L_4 * \sin(\theta_1 + \theta_2 - 360) + L_3 * \sin(\theta_1 - 180) \\ & -P_1 - P_5 \\ & 1 \end{pmatrix} \tag{8}$$

B. Inverse Kinematics

For the resolution of the inverse kinematics, the geometric method was used analyzing only the two links related to the revolute joints because the robot has 2 DOF in the z-axis, which makes infinites solutions. Taking into consideration this problem, an analysis of a 2 DOF robot was performed.

Equation (9) shows the real values of the variables within the diagram.

$$C = L_2 | C_1 = L_3 | b_1 = L_4 \tag{9}$$

Through the geometric method with a base on the calculations carried out in [1], it was necessary to find angle relations that include theta 1 (θ_1) (10) and theta 2 (θ_2) (14) by making tangent relations and convert them into arctangent2 (arctan2), which is a function created for the correct determination of the quadrant of a coordinate (x, y) since arctangent (arctan) only works in quadrant 1 and 4. Inverse cosines and sines were avoided because it is difficult to compute them, making the process slow and affecting the performance of the manipulator.

$$\theta_1 = 90 - \beta_1 - \emptyset_1 \tag{10}$$

To obtain theta 1, each variable was solved using trigonometric relations of a right triangle as shown in (11), (12) and (13).

$$\tan \emptyset_1 = \frac{-y}{x-c} \Rightarrow \emptyset_1 = \arctan 2(-y, x - L_2) \tag{11}$$

$$\tan \beta_1 = \frac{-b_1 * \sin(90-\theta_2)}{c_1 + b_1 * \cos(90-\theta_2)} \beta_1 = \arctan2(-L_4 * \sin(90-\theta_2), L_3 + L_4 * \cos(90-\theta_2)) \tag{12}$$

$$\Rightarrow \theta_1 = 90 - \arctan2(-L_4 * \sin(90-\theta_2), L_3 + L_4 * \cos(90-\theta_2)) - \arctan 2(-y, x - L_2) \tag{13}$$

A trigonometric relationship including theta 2 was sought (14). The relationship was manipulated to be equal to one of the identities of the law of cosines (15). The identity of the law of cosines was used to calculate the sine of theta 2, called D a result as show in (16), (17), (18) and (19).

$$180 = \alpha_1 + (90 - \theta_2) \tag{14}$$

$$\Rightarrow 180 - \alpha_1 = 90 - \theta_2 \Rightarrow \cos(180 - \alpha_1) = \cos(90 - \theta_2) \Rightarrow -\cos\alpha_1 = \sin\theta_2 \tag{15}$$

$$\cos\alpha_1 = \frac{b_1^2 + c_1^2 - a_1^2}{2 * b_1 * c_1} = -\sin\theta_2 \tag{16}$$

$$\sin\theta_2 = \frac{-b_1^2 - c_1^2 + a_1^2}{2 * b_1 * c_1} \tag{17}$$

$$a_1^2 = (x - L_2)^2 + y^2 \tag{18}$$

$$\Rightarrow \sin\theta_2 = \frac{-L_4^2 - L_3^2 + (x - L_2)^2 + y^2}{2 * L_4 * L_3} = D \tag{19}$$

The fundamental trigonometric identity was used to replace sine theta 2 in it and obtain the tangent ratio in (20), (21) and (22). Finally, theta 2 was found using arctan2 as shown in (23).

$$\sin^2\theta_2 + \cos^2\theta_2 = 1 \tag{20}$$

$$\cos\theta_2 = \pm\sqrt{1 - \sin^2\theta_2} \Rightarrow \cos\theta_2 = \pm\sqrt{1 - D^2} \tag{21}$$

$$\frac{\sin\theta_2}{\cos\theta_2} = \frac{D}{\pm\sqrt{1-D^2}} = \tan\theta_2 \tag{22}$$

$$\Rightarrow \theta_2 = \arctan 2\left(D, \pm\sqrt{1-D^2}\right) \tag{23}$$

C. *Direct and inverse kinematics performance in MATLAB*

For the evaluation of the performance of the model, MATLAB was used as primary software solution. The defined parameters and analysis results were defined in the MATLAB workspace in order to study the operation of robot kinematics, Fig. 2. After powering on the robot, it is programmed to always search for and move to its home governed by the location of the end-of-stroke sensors, where its joints are at 0 degrees, seen in fig. 3. However, for the purpose of the simulations only theta 1 and theta was considered, because they are compared with the movements of the inverse kinematics.

```
function [pos,rpy] = SCARA_DOF5(geo,q)
L2=geo(1);
L3=geo(2);
L4=geo(3);

p1=q(1);
q2=q(2);
q3=q(3);
q4=q(4);
p5=q(5);


A01= DH(0,-p1,L2,0);
A12= DH(-pi+q2,0,L3,0);
A23= DH(-pi+q3,0,L4,0);
A34= DH(q4,0,0,0);
A45= DH(0,-p5,0,0);

T=A01*A12*A23*A34*A45;

pos=T(1:3,4)';
[r p y] = mth2rpy(T);
rpy = [r p y];

end
```

Fig. 2 Function in MATLAB for analysis of Inverted SCARA robot of 5 DOF

```
>> [pos,rpy] = SCARA_DOF5([7.87 7.87 7.87],[0 0 0 0 3.94])

pos =

    7.8700    0.0000   -3.9400


rpy =

   1.0e-13 *

         0         0    0.1403
```

Fig. 3 Validation with GEO vector and home position

In fig. 4 it is possible to visualize the validation carried out in the solidworks software, where the possible is equal to the of the mathlab simulation.

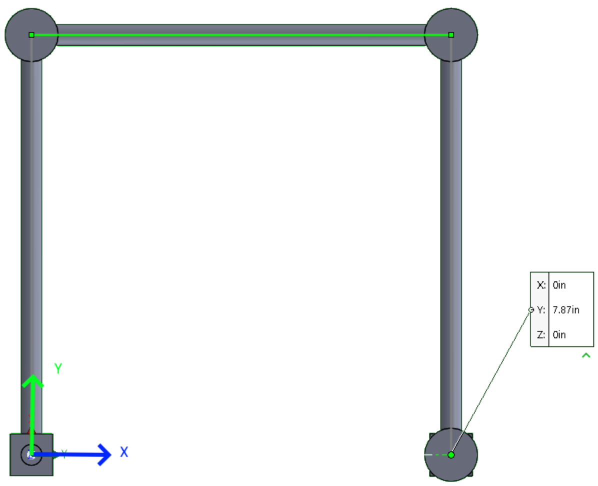

Fig. 4 Validation in solidworks with inverted axes of the 5 DOF robot

V. Conclusions

A successful control system of an inverted SCARA of 5 DOF was obtained using the Denavit-Hartenberg algorithm and a geometric method to solve its kinematics. The direct kinematics of the inverted SCARA of 5 DOF can be solved using the Denavit-Hartenberg algorithm by defining the number of systems and the coordinates of all its joints and links. Since the inverse kinematics of a SCARA of 5 DOF has infinitive solutions due to its 2 DOF in the z-axis, a geometric method can be used considering two rotations joints and their related links, similar to a 2 DOF robot analysis.

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